

UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 1st MAY 2014

Organised by the **United Kingdom Mathematics Trust**
from the **School of Mathematics, University of Leeds**



Institute
and Faculty
of Actuaries

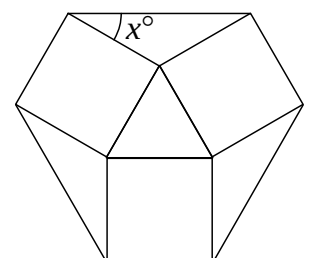
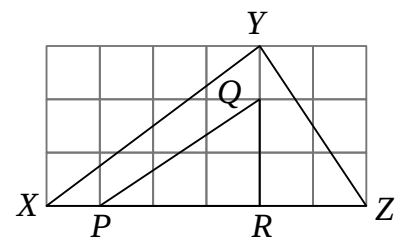
RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 8 or below.
Candidates in Scotland must be in S2 or below.
Candidates in Northern Ireland must be in School Year 9 or below.
5. **Use B or HB pencil only.** Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15.
When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
Six marks are awarded for each correct answer to Questions 16-25.
Each incorrect answer to Questions 16-20 loses 1 mark.
Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options.** The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
The UK JMC is about solving interesting problems, not about lucky guessing.

The UKMT is a registered charity

<http://www.ukmt.org.uk>

1. What is $(999 - 99 + 9) \div 9$?
 A 91 B 99 C 100 D 101 E 109
2. How many minutes are there in $\frac{1}{12}$ of a day?
 A 240 B 120 C 60 D 30 E 15
3. In my row in the theatre the seats are numbered consecutively from T1 to T50. I am sitting in seat T17 and you are sitting in seat T39. How many seats are there between us?
 A 23 B 22 C 21 D 20 E 19
4. The number 987 654 321 is multiplied by 9. How many times does the digit 8 occur in the result?
 A 1 B 2 C 3 D 4 E 9
5. What is the difference between the smallest 4-digit number and the largest 3-digit number?
 A 1 B 10 C 100 D 1000 E 9899
6. The diagram shows a square divided into strips of equal width. Three strips are black and two are grey. What fraction of the perimeter of the square is grey?
 A $\frac{1}{5}$ B $\frac{1}{4}$ C $\frac{4}{25}$ D $\frac{1}{3}$ E $\frac{2}{5}$
7. What is $2014 - 4102$?
 A -2012 B -2088 C -2092 D -2098 E -2112
8. How many prime numbers are there in the list
 1, 12, 123, 1234, 12 345, 123 456 ?
 A 0 B 1 C 2 D 3 E 4
9. Triangles XYZ and PQR are drawn on a square grid. What fraction of the area of triangle XYZ is the area of triangle PQR ?
 A $\frac{1}{4}$ B $\frac{7}{18}$ C $\frac{1}{2}$ D $\frac{5}{18}$ E $\frac{1}{3}$
10. An equilateral triangle is surrounded by three squares, as shown. What is the value of x ?
 A 15 B 18 C 24 D 30 E 36



11. The first two terms of a sequence are 1 and 2. Each of the following terms in the sequence is the sum of all the terms which come before it in the sequence.

Which of these is *not* a term in the sequence?

A 6 B 24 C 48 D 72 E 96

12. In this subtraction, P , Q , R , S and T represent single digits.

$$\begin{array}{r} 7 \quad Q \quad 2 \quad S \quad T \\ - P \quad 3 \quad R \quad 9 \quad 6 \\ \hline 2 \quad 2 \quad 2 \quad 2 \quad 2 \end{array}$$

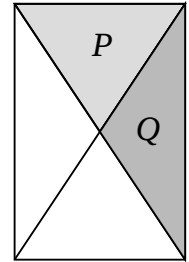
What is the value of $P + Q + R + S + T$?

A 30 B 29 C 28 D 27 E 26

13. A rectangle is split into triangles by drawing in its diagonals. What is the ratio of the area of triangle P to the area of triangle Q ?

A 1 : 1 B 1 : 2 C 2 : 1 D 2 : 3

E the ratio depends on the lengths of the sides of the rectangle

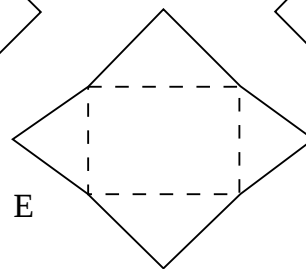
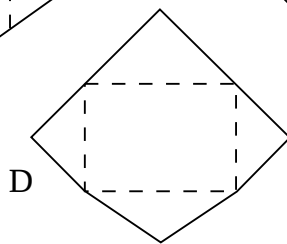
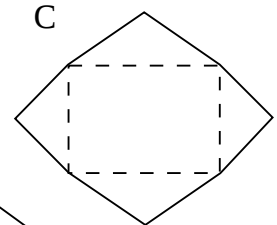
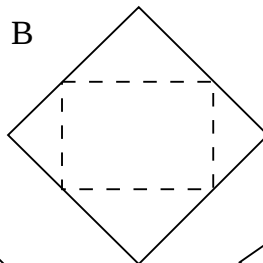
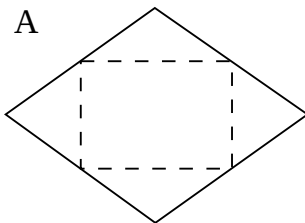
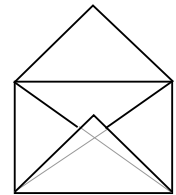


14. Which of these is equal to one million millimetres?

A 1 metre B 10 metres C 100 metres D 1 kilometre E 10 kilometres

15. The diagram shows a rectangular envelope made by folding (and gluing) a single piece of paper.

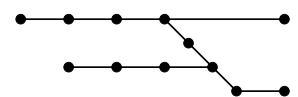
What could the original unfolded piece of paper look like?
(The dashed lines are the fold lines.)



16. Only one of the following statements is true. Which one?

A 'B is true' B 'E is false' C 'Statements A to E are true'
D 'Statements A to E are false' E 'A is false'

17. The diagram is a 'map' of Jo's local rail network, where the dots represent stations and the lines are routes. Jo wants to visit all the stations, travelling only by train, starting at any station and ending at any station, with no restrictions on which routes are taken.



What is the smallest number of stations that Jo must go to more than once?

A 1 B 2 C 3 D 4 E 5

18. Which of these statements is true?

- A $15\,614 = 1 + 5^6 - 1 \times 4$ B $15\,615 = 1 + 5^6 - 1 \times 5$ C $15\,616 = 1 + 5^6 - 1 \times 6$
D $15\,617 = 1 + 5^6 - 1 \times 7$ E $15\,618 = 1 + 5^6 - 1 \times 8$

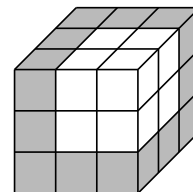
19. Jack and Jill played a game for two people. In each game, the winner was awarded 2 points and the loser 1 point. No games were drawn. Jack won exactly 4 games and Jill had a final score of 10 points. How many games did they play?

- A 5 B 6 C 7 D 8 E impossible to determine

20. Box P has p chocolates and box Q has q chocolates, where p and q are both odd and $p > q$. What is the smallest number of chocolates which would have to be moved from box P to box Q so that box Q has more chocolates than box P?

- A $\frac{q-p+2}{2}$ B $\frac{p-q+2}{2}$ C $\frac{q+p-2}{2}$ D $\frac{p-q-2}{2}$ E $\frac{q+p+2}{2}$

21. Pablo's teacher has given him 27 identical white cubes. She asks him to paint some of the faces of these cubes grey and then stack the cubes so that they appear as shown. What is the largest possible number of the individual white cubes which Pablo can leave with no faces painted grey?

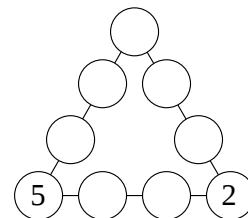


- A 8 B 12 C 14 D 15 E 16

22. In the division calculation $952\,473 \div 18$, which two adjacent digits should be swapped in order to increase the result by 100?

- A 9 and 5 B 5 and 2 C 2 and 4 D 4 and 7 E 7 and 3

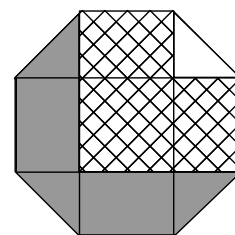
23. Sam wants to complete the diagram so that each of the nine circles contains one of the digits from 1 to 9 inclusive and each contains a different digit. Also, the digits in each of the three lines of four circles must have the same total. What is this total?



- A 17 B 18 C 19 D 20 E 21

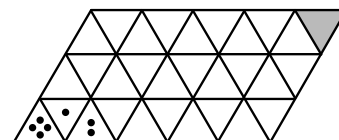
24. The diagram shows a regular octagon with sides of length 1. The octagon is divided into regions by four diagonals.

What is the difference between the area of the hatched region and the area of the region shaded grey?



- A 0 B $\frac{1}{8}$ C $\frac{1}{4}$ D $\frac{1}{2}$ E 1

25. A die has the shape of a regular tetrahedron, with the four faces having 1, 2, 3 and 4 pips. The die is placed with 4 pips 'face down' in one corner of the triangular grid shown, so that the face with 4 pips precisely covers the triangle marked with 4 pips. The



die is now 'rolled', by rotating about an edge without slipping, so that 1 pip is face down. It is rolled again, so that 2 pips are face down, as indicated. The rolling continues until the die rests on the shaded triangle in the opposite corner of the grid. How many pips are now face down?

- A 1 B 2 C 3 D 4 E it depends on the route taken